Noise in homodyne and heterodyne detection

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Quantum-mechanical calculations of the mean-square fluctuation spectra in optical homodyning and heterodyning are made for arbitrary input and local-oscillator quantum states. In addition to the unavoidable quantum fluctuations, it is shown that excess noise from the local oscillator always affects homodyning and, when it is broadband, also heterodyning. Both the quantum and the excess noise of the local oscillator can be eliminated by coherent subtraction of the two outputs of a 50–50 beam splitter. This result also demonstrates the fact that the basic quantum noise in homodyning and heterodyning is signal quantum fluctuation, not local-oscillator shot noise.

Quantum noise in optical heterodyning and homodyning is usually analyzed under the assumption that the photodetector output is conditionally Poisson[1–4] (conditioned on the input signal). This quantum noise is frequently supposed to arise from local-oscillator (LO) shot noise. However, in a fully quantum-mechanical analysis, Yuen and Shapiro[5–7] have shown that it actually arises from the signal quantum fluctuation; in particular, homodyning and heterodyning are realizations of abstract quantum measurements of the field quadratures. This fact permits the use of homodyne detection for probing the small single-quadrature fluctuation in two-photon coherent states[8,9] (TCS's), also called squeezed states.[10] Indeed, one would not be able to observe small TCS field fluctuation by homodyning if homodyne noise were LO shot noise. In this Letter we analyze by a self-contained calculation the effect of LO noise, both quantum and excess noise, on homodyne and heterodyne detection. For conventional input light, i.e., fields in coherent states or their random superpositions, we demonstrate how a conditionally Poisson calculation would yield the correct answer but incorrect interpretation for the detection quantum noise.

The effect of LO excess noise has not been analyzed before. It may limit the performance of a homodyne receiver and, when it is broadband, also that of a heterodyne receiver. Such excess noise is important in connection with a TCS in which a reduced quadrature fluctuation is involved and also with semiconductor-laser LO's currently being investigated for communication applications.[11] We show that, by coherently combining the two beam-splitter outputs, one can eliminate all the LO noise even when the random effects of photodetector quantum efficiency are taken into account.

We first consider homodyning in the configuration of Fig. 1 without photodetector 2 and with bias subtraction before filtering. Let a, b, and c be the photon-annihilation operators corresponding to the signal, the LO, and the photodetector 1 input fields, respectively. By shifting the c field with a constant phase, we have the representation

$$c = \sqrt{\epsilon}a + i\sqrt{1-\epsilon}b. \tag{1}$$

Let $a = a_1 + ia_2$, $b = b_1 + ib_2$, etc. be the quadratures of a, b, etc; a_1, a_2, etc. are Hermitian operators with commutators $[a_1, a_2] = i/2$, $[a_1, b_1] = [a_1, b_2] = 0$, etc. The photon-number operators a^+a, b^+b, etc. are denoted by N_a, N_b, etc. To bring out the essence of the calculation we first ignore the effects of quantum efficiency, detector noise, spectral behavior, and normalization with respect to photocurrent.

We use $\langle\ \rangle$ to denote an average with respect to a quantum state, which may be pure or mixed including excess fluctuation. The mean photodetector output can then be written as

$$\langle N_c\rangle = \epsilon\langle N_a\rangle + (1-\epsilon)\langle N_b\rangle - 2[\epsilon(1-\epsilon)]^{1/2}\langle a_1b_2 - b_1a_2\rangle. \tag{2}$$

For homodyne detection of $\langle a_1\rangle$, we let $\langle b_1\rangle = 0$ so that $\langle a_1\rangle$ is obtained as the mean of A:

$$A \equiv -\{2[\epsilon(1-\epsilon)]^{1/2}\langle b_2\rangle\}^{-1} \times [N_c - (1-\epsilon)\langle N_b\rangle - \epsilon\langle N_a\rangle]. \tag{3}$$

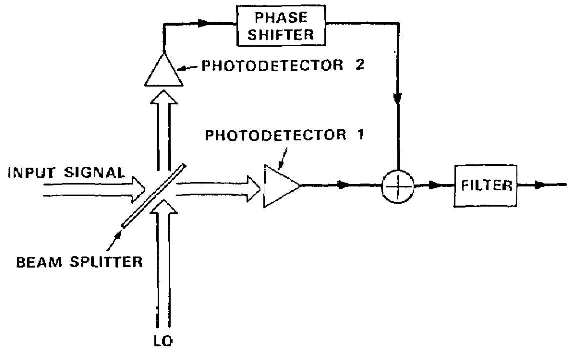

Fig. 1. Schematic for homodyne and heterodyne detection: the beam splitter is assumed lossless with power transmission ϵ and reflection $1-\epsilon$.

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To obtain $\langle A \rangle = \sqrt{\epsilon}\langle a_1 \rangle$, we need the fact that the signal and the LO are independent, viz., $\langle a_1 b_2 \rangle = \langle a_1 \rangle \langle b_2 \rangle$, etc. The mean-square fluctuation $\langle \Delta A^2 \rangle \equiv \langle (A - \langle A \rangle)^2 \rangle$ can be directly computed from Eqs. (1)–(3). The most important step in the computation is

$$\begin{aligned}\langle N_c^2 \rangle = {} & \epsilon^2 \langle N_a^2 \rangle + (1-\epsilon)^2 \langle N_b^2 \rangle \\ & + 2\epsilon(1-\epsilon)\langle N_a \rangle \langle N_b \rangle \\ & + 4\epsilon(1-\epsilon)[\langle b_2^2 \rangle \langle a_1^2 \rangle + \langle b_1^2 \rangle \langle a_1^2 \rangle \\ & - \langle b_1 b_2 \rangle \langle a_2 a_1 \rangle - \langle b_2 b_1 \rangle \langle a_1 a_2 \rangle] \\ & - 2\epsilon[\epsilon(1-\epsilon)]^{1/2} \langle b_2 \rangle (\langle N_a a_1 \rangle + \langle a_1 N_a \rangle) \\ & - 2(1-\epsilon)[\epsilon(1-\epsilon)]^{1/2}[\langle a_1 \rangle (\langle N_b b_2 + b_2 N_b \rangle) \\ & - \langle a_2 \rangle (\langle N_b b_1 \rangle - \langle b_1 N_b \rangle)]. \end{aligned} \tag{4}$$

Most of the noise contribution from Eq. (4) can be suppressed with large $\langle N_b \rangle$. To locate the essential noise terms, we let $\langle N_b \rangle \to \infty$, with $\langle N_b \rangle / \langle b_2 \rangle^2 \to 1$. This means that the LO power comes primarily from its mean quadrature $\langle b_2 \rangle$ and is large compared with the other moments and cross moments of a and b up to the fourth order. After straightforward algebra, we obtain

$$\langle \Delta A^2 \rangle = \epsilon \langle a_1^2 \rangle + (1-\epsilon)\langle \Delta N_b^2 \rangle / 4 \langle N_b \rangle. \tag{5}$$

The signal quadrature fluctuation $\langle \Delta a_1^2 \rangle$ depends on the quantum state of the input light. Since $\langle \Delta N_b^2 \rangle \geq \langle N_b \rangle$ is characteristic of photon-number fluctuation,[12] the second term above does not vanish as $\langle N_b \rangle$ gets large. However, ϵ can in principle be made arbitrarily close to 1; thus homodyning sees the signal quadrature fluctuation, as first proved in Refs. 5–7.

To examine the LO-noise contribution more closely, let us assume that in addition to its large mean field $\langle b_2 \rangle$ the LO has an additive Gaussian excess noise so that $\langle N_b \rangle = \langle b_2 \rangle^2 + N$. In this case,

$$\langle \Delta N_b^2 \rangle \equiv \langle N_b^2 \rangle - \langle N_b \rangle^2 = \langle b_2 \rangle^2 (1 + 2N) + N. \tag{6}$$

The easiest way to obtain Eq. (6) is to compute the moments of $b' = b + n$, where the b field is in a coherent state with parameter $i\langle b_2 \rangle$ and n is an ordinary complex Gaussian random variable with variance N. For $N/\langle b_2 \rangle^2 \sim 0$, Eq. (5) becomes

$$\langle \Delta A^2 \rangle = \epsilon \langle \Delta a_1^2 \rangle + (1-\epsilon)(1/4 + N/2). \tag{7}$$

As would be expected intuitively, the signal noise enters $\langle \Delta A^2 \rangle$ through the beam-splitter transmissivity ϵ and the LO noise through the reflectivity $1 - \epsilon$. The LO shot noise or quantum noise of 1/4 and excess noise $N/2$ are thus not suppressed by LO power, just as the signal noise cannot be so suppressed. Since ϵ can never be exactly unity, the detection scheme is not quantum limited whenever the LO-noise term of Eq. (7) is larger than $\epsilon\langle \Delta a_1^2 \rangle$. This situation may occur when we wish to observe a small $\langle \Delta a_1^2 \rangle$ in a TCS and also when $(1-\epsilon)N$ is large, as in semiconductor-laser LO's.[11]

To account for the effect of quantum efficiency $\eta < 1$, the representation $c' = \eta^{1/2} c + (1-\eta)^{1/2} c''$ for a vacuum c'' field should be used[5–7] in lieu of c. The spectrum $\langle \Delta A^2(\omega) \rangle$ of $A(t)$ can be obtained through the correlation function $\langle A(t) A(0) \rangle$, which in turn can be computed similarly to the calculation of Eq. (5). Let $\bar{I}(t) = eG\eta \langle N_c(t) \rangle / \hbar\omega_0$ be the mean photocurrent, where ω_0 is the nominal frequency of the fields and G is the nonrandom photodetector gain. The detector thermal noise $2kT/R$ and dark current eGI_D can be added in the standard fashion.[3,4] Let $P_{\mathrm{LO}} = \hbar\omega_0 \langle N_b \rangle$ be the LO power incident upon the beam splitter. We look at the normalized photocurrent $I_s(t) = I(t)\hbar\omega_0/eG\eta 2[(1-\epsilon)P_{\mathrm{LO}}]^{1/2}$ whose mean is $\overline{I_s(t)} = \sqrt{\epsilon}\langle a_1(t) \rangle$ + bias. Its spectrum is obtained by a scaling of $\langle \Delta A^2(\omega) \rangle$,

$$\begin{aligned}\overline{\Delta I_s^2(\omega)} = {} & \epsilon \langle \Delta a_1^2(\omega) \rangle \\ & + \frac{1-\eta}{4\eta} + \frac{1-\epsilon}{4} \frac{\langle \Delta N_b^2(\omega) \rangle \hbar\omega_0}{P_{\mathrm{LO}}} \\ & + \frac{\epsilon^2}{4(1-\epsilon)} \frac{\langle \Delta N_a^2(\omega) \rangle \hbar\omega_0}{P_{\mathrm{LO}}} \\ & + \frac{2kT/R + eGI_D}{4\eta(1-\epsilon)P_{\mathrm{LO}}(eG)^2/\hbar\omega_0}. \end{aligned} \tag{8}$$

The first term gives the signal-quadrature fluctuation spectrum including both quantum and classical noises. The second term represents the vacuum noise contribution[5–7] for nonunity quantum efficiency. The LO noise enters the third term through the LO photon-number fluctuation spectrum, which includes LO shot noise. The suppressed-signal photon fluctuation spectrum is also displayed in the fourth term. It is seen that the power suppressing $\langle \Delta N_b^2 \rangle$ is P_{LO}, whereas $\langle \Delta N_a^2 \rangle$ and the detector noise are suppressed by $(1-\epsilon)P_{\mathrm{LO}}$. Thus the requirement of device-noise suppression may limit the value of ϵ away from unity, in conflict with the goal of LO-noise suppression.

For coherent conventional input lights in the absence of LO excess noise, we have the Poisson result $\overline{I^2} = \bar{I} \simeq \eta/\hbar\omega_0(1-\epsilon)P_{\mathrm{LO}}$ in the quantum limit. After normalization to I_s, this gives $\overline{I_s^2} = 1/4\eta$, in agreement with the above quantum result with $\langle \Delta a_1^2 \rangle$. Since conditionally Poisson calculations are quantum mechanically valid when both the signal and the LO fields are conventional lights, this agreement is expected. Indeed, the full spectrum [Eq. (8)] also agrees with a conditionally Poisson result if the LO is also a conventional source. However, as is evident from Eq. (5) or (8), the conditionally Poisson result does not yield the correct answer when the signal exhibits nonconventional behavior like TCS's with $\langle \Delta a_1^2 \rangle < 1/4$. This can be traced to the fact that the photocount statistics in such a case is, quantum mechanically, *not* conditionally Poisson.[8] Thus Eq. (5) or (8) must be employed in place of the conditionally Poisson result. Even for conventional light, the interpretation of detection quantum noise as LO shot noise, which seems natural from the $\overline{I^2} = \bar{I}$ calculation above, is nevertheless *incorrect* quantum mechanically. From Eq. (5) it can be seen that the LO shot noise provides only a small contribution to the detection quantum noise, vanishing in the limit $\epsilon \to 1$. The dominant quantum noise comes from the signal, in fact from the term $4\epsilon(1-\epsilon)\langle b_2^2 \rangle \langle a_1^2 \rangle$ in Eq. (4), which is a cross term between the signal and the LO.

Heterodyning can be treated similarly. In this case the image band plays an important role,[7] so that instead of Eq. (1) we have

$$\begin{aligned} c = {} & \sqrt{\epsilon}\{a \exp[-j(\omega_0 + \omega_{IF})t] + a' \\ & \times \exp[-j(\omega_0 - \omega_{IF})t]\} + i\sqrt{1-\epsilon}\, b \exp(-j\omega_0 t). \end{aligned} \tag{9}$$

In Eq. (9) we display the dominant oscillations of the fields explicitly, with the image-band field a' at frequency $\omega_0 - \omega_{IF}$ when the signal is at frequency $\omega_0 + \omega_{IF}$. The photocurrent quadratures yield the signal quadratures, of which we look at only one for simplicity. First ignoring other effects, we let

$$A_1 \equiv \frac{1}{[(1-\epsilon)P_{\mathrm{LO}}/\hbar\omega_0]} \int N_c(t) \cos\omega_{IF}t\,dt = \left(\frac{\epsilon\hbar\omega_0}{P_{\mathrm{LO}}}\right)^{1/2}(a_1b_2 - b_1a_2 + a'_1b_2 - b_1a'_2). \quad (10)$$

With $\langle a'_1\rangle = \langle b_1\rangle = 0$, we have $\langle A_1\rangle = \sqrt{\epsilon}\langle a_1\rangle$. In contrast to homodyning, the N_a and N_b terms do not appear in A_1 since they have been filtered away; thus the photon fluctuations do not contribute to $\langle \Delta A_1^2\rangle$. Indeed, in the large P_{LO} limit we have instead of Eq. (5)

$$\langle \Delta A_1^2\rangle = \epsilon[\langle \Delta a_1^2\rangle + \langle \Delta a'^2_1\rangle], \quad (11)$$

which contains *no* contribution from the LO noise at all. For the a' field in vacuum state, as it should ideally be for heterodyning, $\langle \Delta A_1^2\rangle = \epsilon[\langle \Delta a_1^2\rangle + 1/4]$, which gives the correct $\langle \Delta A_1^2\rangle = \epsilon/2$ for coherent-state input light. In general, the signal and image-band quadrature fluctuations provide the fundamental noise limit in heterodyne detection.

To include other effects, we assume as usual that the signal bandwidth is much smaller than ω_{IF}. Let $I_1(t) \equiv I'(t)\hbar\omega_0/eG\eta[(1-\epsilon)P_{\mathrm{LO}}]^{1/2}$, where $I'(t)$ is the filtered photocurrent. Then $\overline{I_1(t)} = \sqrt{\epsilon}\langle a_1(t)\rangle$ and

$$\begin{aligned}\overline{\Delta I_1^2(\omega)} &= \epsilon[\langle \Delta a_1^2(\omega)\rangle + \langle \Delta a'^2_1(\omega)\rangle] + \epsilon\frac{1-\eta}{2\eta} \\ &+ \frac{1-\epsilon}{2}\frac{\langle \Delta N_b^2(\omega+\omega_{IF})\rangle\hbar\omega_0}{P_{\mathrm{LO}}} \\ &+ \epsilon^2\frac{\langle \Delta N_a^2(\omega+\omega_{IF})\rangle\hbar\omega_0}{(1-\epsilon)P_{\mathrm{LO}}} \\ &+ \frac{2kT/R + eGI_D}{2\eta(1-\epsilon)P_{\mathrm{LO}}(eG)^2/\hbar\omega_0}. \end{aligned} \quad (12)$$

The photon fluctuation, namely, the third and fourth terms of Eq. (12), contributes to $\overline{\Delta I_1^2(\omega)}$ only when its bandwidth is greater than ω_{IF}. This situation occurs in semiconductor lasers.[1] In particular, the spontaneous-emission contribution to photon-number fluctuation in lasers is always broadband.

As the absence of LO noise in Eq. (11) can be traced to the absence of N_b in Eq. (10), it appears that the LO-noise term in Eqs. (5), (8), and (12) can be eliminated by coherently subtracting the current outputs of photodetectors 1 and 2 of Fig. 1. For this purpose we use $\epsilon = 1/2$ and the same quantum efficiency for both photodetectors. If d is the input field to photodetector 2, we have in homodyning with arbitrary η

$$d = a/\sqrt{2} - ib/\sqrt{2}, \quad (13)$$

$$c' = \eta^{1/2}c + (1-\eta)^{1/2}c'',$$

$$d' = \eta^{1/2}d + (1-\eta)^{1/2}d'', \quad (14)$$

where a constant phase shift has been added to d and the c'', and d'' fields are in the vacuum state. Now $\langle A'\rangle = \langle a_1\rangle$ for

$$A' \equiv (N_{c'} - N_{d'})/2\eta\left(\frac{P_{\mathrm{LO}}}{\hbar\omega_0}\right)^{1/2}. \quad (15)$$

With large P_{LO}, the mean-square fluctuation becomes

$$\langle \Delta A'^2(\omega)\rangle = \langle \Delta a_1^2(\omega)\rangle + (1-\eta)/4\eta. \quad (16)$$

For conventional light, this result can also be derived using a Poisson model. Note that, in addition to eliminating the LO noise, we have also recovered all the signal energy. In heterodyning, the same procedure eliminates the N_a, N_b photon fluctuation terms of Eq. (12). This result is not affected by a more-careful analysis of the randomness effects associated with photodetector quantum efficiency. Equation (16) demonstrates clearly that it is the signal fluctuation that fundamentally limits a homodyne receiver.

This LO-noise-cancellation scheme may also find significant applications in coherent optical fiber systems. In fact, it is the optical analog of the microwave balanced-mixer radiometer.[13] However, there are certain differences between the microwave and the optical cases. In particular, the photodetector output is a random point process even for a fixed coherent input. Nevertheless, the above analysis shows that this scheme works even if the system is in an arbitrary quantum state.

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11. See, e.g., V. W. S. Chan, Proc. Soc. Photo. Opt. Instrum. Eng. **295**, 10 (1981); Y. Yamamoto and T. Kimura, IEEE J. Quantum Electron. **QE-17**, 919 (1981).
12. Note the distinction between the quadrature fluctuation $\langle \Delta a_1^2\rangle$ and the photon number fluctuation $\langle \Delta N_a^2\rangle$ of a field. The photon shot noise is the factor $\langle N_a\rangle$ in $\langle \Delta N_a^2\rangle = \langle N_a\rangle + (\langle a^{+2}a^2\rangle - \langle N_a\rangle^2)$, the second term being the intensity fluctuation.
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