Applied Physics 114c: Homework #3

(Dated: April 29, 2016)

Due: Monday, May 9th (box outside Watson 264, anytime before midnight)

Note: In the following problems we will use the notation **B** for the macroscopic spatially averaged magnetic flux density (or magnetic induction) in a superconductor, **H** for the magnetic field, and $\mathbf{h}(\mathbf{r})$ for the microscopic magnetic flux density in the superconductor (as per Tinkham). We will also use cgs units.

1. **Reading**

   Chapter 1 and 2 of Tinkham, and assorted reference papers provided on the class website.

2. **(10 points) Thin flat slab in an applied parallel magnetic field.** Consider a thin slab of (type-I) superconductor of thickness d in an applied parallel magnetic field H_a.

   (a) Using the London equations, solve for the microscopic magnetic flux density ($\mathbf{h}$) as a function of depth within the slab (here depth is taken along coordinate x, and the slab region occurs between $x = \pm d/2$). State any assumptions you make.

   (b) Show that for $d \ll \lambda$ one gets non-perfect diamagnetism with average magnetization density $M \approx -(H_a/4\pi)(d^2/12\lambda^2)$ within the superconductor.

   (c) Show that the critical field for such a thin slab is much larger than that of the thermodynamic critical field H_c.

3. **(10 points) Demagnetization factor and intermediate state for a superconducting sphere.** Recall that for macroscopic Maxwell's equations we have continuity of the tangential components of the magnetic field (H_{tang}) and the normal component of the magnetic flux density (B_n). H_c is the critical magnetic field in which the normal and

superconducting state can coexist in equilibrium.

(a) Derive the demagnetization factor, η, for a superconducting sphere of radius $R \gg \lambda$ and for applied uniform magnetic field $H_a \ll H_c$.

(b) For applied fields $(1-\eta)H_c < H_a < H_c$ the sphere will be in an "intermediate state" with both superconducting and normal states coexisting. For an ideal type-I superconductor (and assuming $H \approx H_c$ in the normal regions), solve for the macroscopically averaged magnetic flux density in the sphere in the intermediate state. Comment on any assumptions you make about the H field within the sphere, in particular across the boundary between the normal (N) and superconducting (S) states laminae that form within the sphere.

4. **(10 points) Resistance of a superconducting wire above its critical current.** Consider a long superconducting wire of cross-sectional radius $a \gg \lambda$.

   (a) Use the London equations and Maxwell's equations to show that current, I, carried by the wire will be transported in a thin surface layer of thickness λ.

   (b) Argue then that the critical current at which point the wire will begin to become resistive is equal to $I_c = caH_c/2$. For a thinner wire in which $a \ll \lambda$ show (argue) that the critical current can be much lower than this value.

   (c) For $I > I_c$ why can't the surface go normal, leaving a residual superconducting wire core (and no resistance)? Why doesn't the whole wire just go normal?

   (d) Argue for a certain geometrical form for the intermediate state. Assuming this intermediate state of the wire for $I > I_c$, derive the fractional resistance R/R_n (where R_n is the resistance of the wire in its normal state) as a function of I for $I > I_c$.