Deterministic and Nondestructively Verifiable Preparation of Photon Number States

JM Geremia*

Quantum Measurement & Control, Department of Physics and Astronomy, The University of New Mexico, Albuquerque, NM 87131, USA

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An experimentally viable approach for preparing arbitrary photon number states of a cavity mode using continuous measurement and real-time quantum feedback is presented. The procedure passively monitors the number state actually achieved in each feedback-stabilized measurement trajectory, thus providing nondestructively verifiable photon generation. The feasibility of a possible cavity QED implementation in the many-atom, good-cavity-coupling regime is analyzed.

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Practical schemes for preparing number states of the electromagnetic field are sought for the enabling role these states play in quantum information science. For example, single photons provide a valuable resource that can be used in quantum cryptography, communication, and quantum computation with linear optics [1,2]. Multiphoton number states are desired for their role in realizing subshot noise quantum metrological procedures and as a reagent for synthesizing complex nonclassical fields, including optical Schödinger cat states [3].

With these information-theoretic applications in mind, one can compile a list of features desired for ideal number state preparation: (1) *determinism*, meaning that the target photon number is produced with high probability in every state-preparation shot, (2) *verifiability*, meaning that the number of photons actually generated can be diagnosed nondestructively in every shot, and (3) *extendability*, meaning that a single device can easily vary the photon number from shot to shot. Different applications benefit from these disparate features in different ways. For instance, verifiability is likely the highest priority for cryptographic security. High determinism, on the other hand, offers improved efficiency for quantum communication and computation. Ideally, a single physical device could meet all three objectives simultaneously.

To date, quality single photons have been produced in trapped-atom cavity QED experiments [4], quantum dots [5,6], ballistic-atom cavity QED [7], and collective excitations of an atomic ensemble [8,9]. A theoretical approach for extending cavity QED schemes to higher photon numbers by capitalizing on collective dark states found in the symmetric group of multiple atoms has been suggested [3] and two-photon states of a micromaser have been dynamically generated [10]. But while highly deterministic, trapped-atom cavity QED experiments demand the strong cavity-coupling regime and require *exactly* N intracavity atomic excitations to achieve an N-photon number state [3]. Varying the photon (atom) number in back-to-back shots in this regime is likely to be difficult. It is not yet understood whether quantum dot, ballistic-atom cavity QED, or atomic ensemble schemes can be reasonably extended to higher photon numbers. None of these procedures offer inherent single-shot verifiability.

Here, we introduce a procedure for preparing cavity number states that is simultaneously deterministic, intrinsically verifiable, and naturally capable of producing arbitrary photon numbers. The procedure is based on a continuous photon number measurement [11] embedded within a real-time quantum feedback control loop [12,13]. Basic quantum mechanics specifies that conditioning the state of the cavity field on the outcome of a photon number measurement reduces the field to a measurement eigenstate (or at least an approximate eigenstate in practice) [14]. Quantum feedback renders this state reduction process deterministic by actively stabilizing the measurement outcome to the target photon number eigenstate with arbitrarily high probability [15]. With a properly designed feedback policy, any possible measurement outcome (number eigenstate) is a viable candidate for stabilization in every distinct measurement trajectory. And, since the state preparation is performed by a nondestructive quantum measurement, passive single-shot verifiability is an unavoidable fringe benefit of the procedure's own internal anatomy.

Figure 1 provides a schematic of the feedback-stabilized measurement analyzed below. Continuous observation of the photon number, n, in a single cavity mode, $\hat{a}$, is implemented by coupling that cavity mode to an auxiliary probe field, $\hat{b}$, via a cross-Kerr nonlinear scattering Hamiltonian, $\hat{H}_{\rm int} = \hbar\chi\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}$ (χ is the strength of the nonlinearity). For a coherent state probe, the mode-

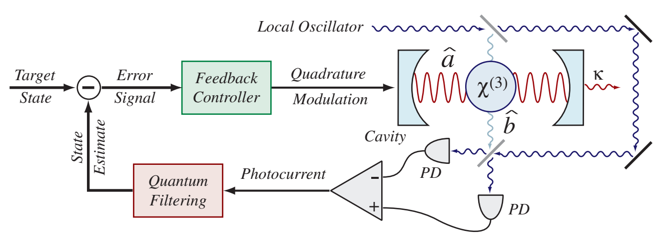

FIG. 1 (color online). Schematic of a feedback-stabilized continuous measurement of cavity photon number.

coupling interaction induces a phase shift that is proportional to the cavity photon number, n [16]. Thus, observing the probe's phase provides an indirect measurement of the cavity number, n.

Such a measurement can be implemented, as in Fig. 1, by placing the Kerr nonlinearity in one arm of a Mach-Zender interferometer and performing balanced homodyne detection on the forward-scattered probe field. The output of the homodyne detectors in this configuration is given by the continuous photocurrent [12,17],

$$dy_t = 2\eta\sqrt{M}ndt + \sqrt{\eta}dW_t, \tag{1}$$

where η is the quantum efficiency of the photodetectors, M is a rate referred to as the *measurement strength* (described in detail below), and the dW_t are Gaussian stochastic increments that reflect quantum noise in the continuous measurement.

Prior to conducting any feedback, these quantum fluctuations must be filtered from the photocurrent to obtain an optimal time-dependent estimate of the cavity photon number [13,17]. The problem of extracting such an estimate is the subject of *quantum filtering theory* [15,18], a field that combines elements from classical signal processing and stochastic analysis with the theory of continuously observed open quantum systems. Here, quantum filtering is conducted by propagating the time-dependent cavity state, $\hat{\rho}_t$, according to a master equation,

$$\begin{aligned} d\hat{\rho}_t = & -i[\hat{H}_t^{(\mathrm{fb})}, \hat{\rho}_t]dt + M\mathcal{D}[\hat{n}]\hat{\rho}_t dt + \kappa\mathcal{D}[\hat{a}]\hat{\rho}_t dt \\ & + \sqrt{M}\mathcal{H}[\hat{n}]\hat{\rho}_t(dy_t - 2\eta\sqrt{M}\mathrm{Tr}[\hat{n}\hat{\rho}_t]dt), \end{aligned} \tag{2}$$

that conditions it on the information provided by the accumulating measurement data, dy_t. The superoperators in Eq. (2) are given, as usual, by $\mathcal{D}[\hat{r}]\hat{\rho}_t \equiv \hat{r}\hat{\rho}_t\hat{r}^\dagger - \frac{1}{2}\times(\hat{r}^\dagger\hat{r}\hat{\rho}_t + \hat{\rho}_t\hat{r}^\dagger\hat{r})$ and $\mathcal{H}[\hat{r}]\hat{\rho}_t \equiv \hat{r}\hat{\rho}_t + \hat{\rho}_t\hat{r}^\dagger - \mathrm{Tr}[(\hat{r} + \hat{r}^\dagger)\hat{\rho}_t]\hat{\rho}_t$. The first term in the master equation describes any Hamiltonian driving (such as feedback) performed on the system, the second term describes decoherence caused by coupling the cavity mode to the probe, the third term reflects cavity decay through the mirrors, and the final term conditions the state on the measurement via the innovation process, $dy_t - 2\eta\sqrt{M}\langle\hat{n}\rangle_t dt$.

The optimal photon number estimate at time t is obtained from the continuously conditioned cavity state as $\langle\hat{n}\rangle_t \equiv \mathrm{Tr}[\hat{n}\hat{\rho}_t]$. Equation (2) thus implicitly provides the crucial feedback ingredient known as the *error signal*,

$$e_t = n^\star - \langle\hat{n}\rangle_t, \tag{3}$$

computed as the deviation of the estimated photon number from the target, $n^\star$. Feedback can then be performed by driving the cavity in response to the error signal. The following analysis considers a feedback control policy,

$$\hat{H}_t^{(\mathrm{fb})} = \tfrac{1}{2}Ge_t(\hat{a} + \hat{a}^\dagger), \tag{4}$$

that drives the cavity amplitude quadrature, $\hat{X} \equiv \frac{1}{2}\times(\hat{a} + \hat{a}^\dagger)$, in proportion to the continuous error signal, e_t, with dc loop gain, G. This feedback policy has the satisfying intuitive interpretation that the controller will increase the amplitude of the intracavity field when its current estimate of the cavity photon number, $\langle\hat{n}\rangle_t$, is below that of the target, $n^\star$, and *vice versa*. Equation (4) also highlights the stochastic nature of quantum feedback, which unlike classical servos typically encountered in the physics laboratory, demands a quantum filter instead of, say, an integrator to achieve feedback stability.

As in any cavity QED situation, true number states are only achieved in the idealized limit where $\kappa \to 0$. This limit provides an illustrative demonstration of the salient features of feedback-stabilized number state preparation (realistic parameters considered below). Figure 2 depicts a simulated ideal measurement where the objective is to prepare an $n^\star = 2$ number state beginning from vacuum.

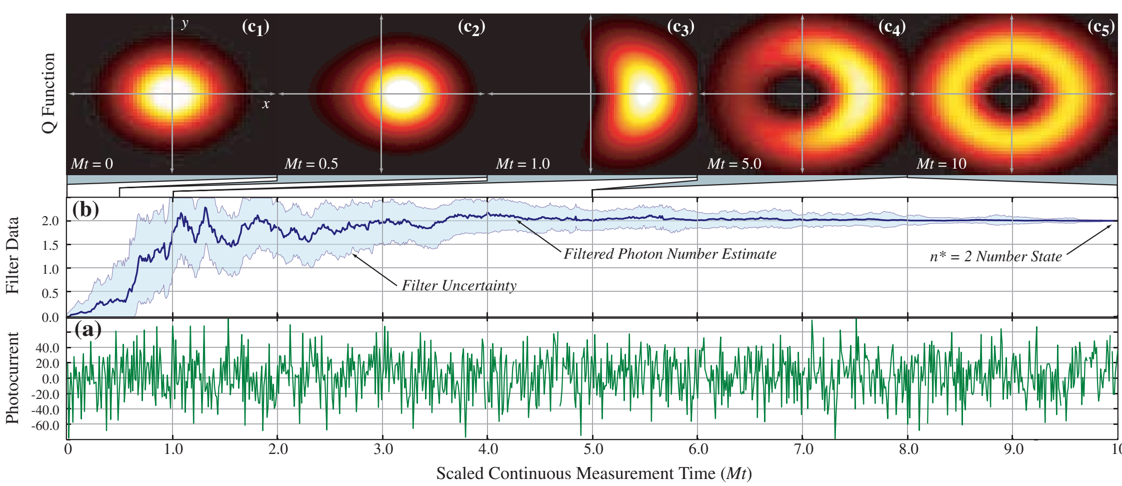

FIG. 2 (color online). An idealized photon number measurement ($\kappa \to 0$, $\eta = 1$, other parameters from Table I) where filtering the continuous photocurrent (a) provides an optimal real-time estimate (b) of the cavity photon number used to drive the measurement to a deterministic outcome via feedback. The evolving cavity mode Q function (c) illustrates the process.

The homodyne photocurrent, y_t in Fig. 2(a), is clearly swamped with quantum noise. However, by propagating the quantum filtering equation [19] subject to y_t as more data become available, the controller extracts its optimal real-time estimate of the cavity photon number [Fig. 2(b)]. Uncertainty in the estimate [shaded region in Fig. 2(b)] is gradually reduced by the measurement.

Figures 2(c)1, 2(c)2, 2(c)3, 2(c)4, and 2(c)5 highlight the feedback-mediated progression to the target eigenstate by depicting the cavity Q function as it changes in time ($Q_t(\alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}_t|\alpha\rangle$ is a quasiprobability distribution parametrized by the coherent state amplitude, $\alpha = x + iy \in \mathbb{C}$). Beginning from the vacuum Q function, $Q_0(\alpha) = \frac{1}{\pi} \times \exp(-\frac{1}{4}|\alpha|^2)$ in Fig. 2(c)1, the quantum filter rapidly finds that the photon number is small relative to the target $n^* = 2$. Feedback consequently displaces the cavity mode toward $\langle\hat{n}\rangle \sim 2$ by driving the in phase to increase the intracavity field, seen in Fig. 2(c)2 at time $Mt \sim 0.1$. Conversely, when $n_t > n^*$, the cavity is driven out of phase to decrease its field. As the measurement proceeds, the photon number uncertainty, $\langle\Delta\hat{n}\rangle$, begins to decrease at the expense of phase uncertainty [Fig. 2(c)3], eventually producing a heavily number-squeezed state [Fig. 2(c)4 at $Mt \sim 5$]. The target $n^* = 2$ eigenstate—with its telltale phase-delocalized Q function—is ultimately achieved [Fig. 2(c)5 at $Mt = 10$].

Of course, a viable laboratory implementation of the cross-Kerr nonlinear optical Hamiltonian [16] is needed for feedback to be practical. A suitable interaction is provided by an atomic dark-state mechanism analogous to that proposed for giant free space Kerr nonlinearities [20]. We consider a sample of N intracavity atoms with the hyperfine level structure depicted in Fig. 3 where two nonradiative hyperfine stretched states are coherently coupled via a two-photon transition that involves both the cavity mode and a strong drive laser. A photocurrent of the form in Eq. (1) is obtained by dispersively coupling the probe field to the atomic hyperfine structure.

In the limit where atomic motion can be neglected, as would be the case for trapped intracavity atoms, the measurement strength is found to be

$$M = \frac{P}{\hbar\omega}\left[\frac{3N\Gamma\lambda^2}{4\pi^2 r^2\Delta}\left(\frac{g_0^2}{g_0^2+\Omega^2}\right)\right]^2, \tag{5}$$

for probe power P, frequency ω (wavelength λ), and detuning Δ. Here, Γ is the atomic spontaneous emission rate, r is the radius of the atomic sample, g_0 is the single photon cavity-coupling rate, and Ω is the Rabi frequency associated with the drive laser.

We envisage trapping $N \sim 1 \times 10^6$ Cs atoms within the mode volume of a $L \sim 4$ cm Fabry-Perot cavity with mirror finesse, $\mathcal{F} \sim 3 \times 10^5$ and ROC $= 25$ cm. Parameters derived from these cavity properties are listed in Table I, and the measurement strength is found to be $M \sim 2.5$ MHz compared to a cavity decay rate of $\kappa \sim 12$ kHz. This corresponds to the many-atom cavity QED strong-coupling

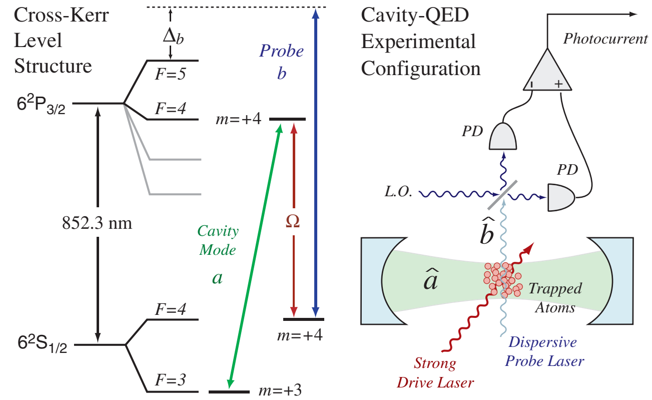

FIG. 3 (color online). Cavity QED implementation of the continuous photon number measurement.

regime, $Ng_0^2/\Gamma\kappa \gg 1$. Given that the total atomic scattering rate for these parameters is $\gamma_s \sim 2.5$ kHz, spontaneous emission due to the probe can be reasonably neglected as fewer than 100 atoms will be scattered on average in the measurement time.

Figure 4 demonstrates the level of performance that we expect from the feedback-stabilized measurement given the parameters in Table I. Five typical simulated measurement trajectories are shown in Fig. 4(a)1. Comparing the Q functions in Figs. 4(a)2 and 4(a)3 with those in Fig. 2 suggests that there is little qualitative difference in this measurement relative to the ideal $\kappa = 0$ case. In general, we expect that $M \gg \kappa$ will be needed such that the measurement can reduce the cavity mode to a good approximate number state prior to appreciable decay.

A quantitative analysis of the feedback stability was conducted by computing the distance of the cavity state from the target eigenstate as a function of time. To do so, we employed the following distance metric,

$$\mathfrak{D}[Q_t] = 1 - \frac{1}{\pi 4^{(n^\star+1)} n^\star!}\int_{\mathbb{C}} |\alpha|^{2n^\star} e^{(-1/4)|\alpha|^2} Q_t d\alpha. \tag{6}$$

Note that $\mathfrak{D}$ is a scalar functional from the space of Q functions into the real numbers between zero and one,

TABLE I. Simulation parameters used to analyze experimental feasibility of a cavity QED feedback implementation.

Parameter	Symbol	Value	Units
Probe power	P	1	μW
Probe wavelength	λ	852.35	nm
Probe detuning	Δ	2	GHz
Cavity decay rate	κ	12	kHz
Cavity-coupling rate	g_0	200	kHz
Atom sample radius	r	110	μm
Drive laser intensity	I	1/4	I_{sat}
Feedback dc gain	G	20	dB
Detector efficiency	η	80	%

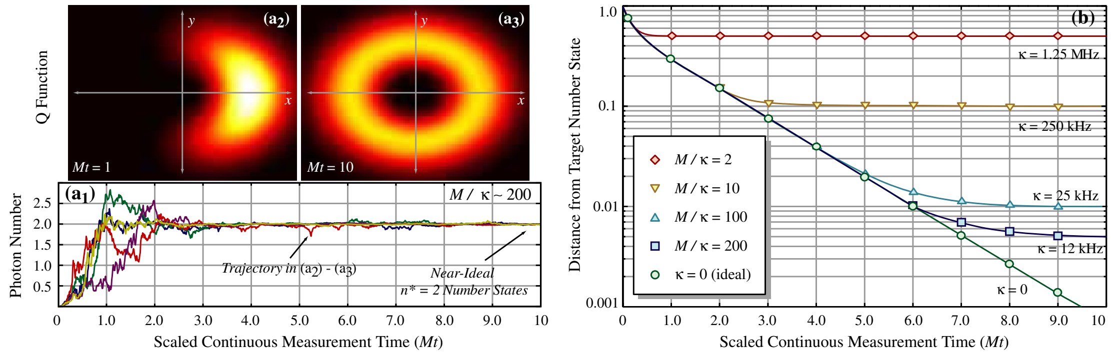

FIG. 4 (color online). Simulated feedback-stabilized photon number measurement trajectories (a) generated using the parameters listed in Table I and feedback stability (b) for different cavity decay rates (solid lines depict theoretical results).

$\mathfrak{D}[\cdot]: L_2(\mathbb{C}) \rightarrow \mathbb{R}_{[0,1]}$. Moreover, $\mathfrak{D}$ assumes the value $\mathfrak{D}[Q_t] = 0$ only when the cavity is in the target number state, its maximum value $\mathfrak{D}[Q_t] = 1$ when the cavity is in any other number state and smoothly interpolates otherwise. Also note that $\mathfrak{D}[\cdot]$ is related closely to the fidelity of the cavity mode with respect to the target number state as $F(\hat{\rho}_t) \approx 1 - \mathfrak{D}[Q_t]$. We find that the distance between the cavity state and the target eigenstate, as measured by $\mathfrak{D}$, strictly decreases in expectation with time, $\mathfrak{D}\mathbb{E}[\mathfrak{D}[Q_t]] \leq 0$. Figure 4(b) illustrates the time evolution of the distance measure averaged over 10^5 trajectories for different cavity decay rates. The experimental parameters in Table I correspond to $M/\kappa \sim 200$, which leads to an $n^\star = 2$ number state with better than 99% fidelity. As expected, the quality of the state preparation is degraded for high cavity decay, with $F \sim 50\%$ for $M = \kappa$.

Given these findings, quantum feedback stabilization of a continuous cavity photon number measurement will likely provide a practical route to heralded production of arbitrary deterministic photon number states. The procedure is anticipated to be robust to reasonable uncertainty in the intracavity atom number, as each individual atom is only weakly coupled to the cavity and fluctuations in M are feedback suppressed. Thus, hidden deterministic prerequisites, such as having to trap exactly N^* atoms to produce the number state with $n^* = N^*$ photons [3,4], are avoided. Our feedback procedure will almost surely rival deterministic cavity QED single photon sources [4] while offering verifiability and less demanding cavity coupling. High-fidelity extraction of $n > 1$ number states is currently being investigated, without neglecting the importance of intracavity number states for metrology and quantum information science.

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*Electronic address: jgeremia@unm.edu

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