Applied Physics 190c: Homework #2

(Dated: April 20, 2017)

Due: Friday, April 28th

1. **(60 points) Measuring squeezing using a Balanced Homodyne Detector**

Assume here that you have access to an ideal balanced homodyne detector with unit quantum efficiency detectors and perfect balancing. Consider an input signal which is generated from the output of a non-degenerate parametric down conversion (NDPDC) process in a nonlinear crystal. In this situation we create correlations between the output fields at δ (signal) and $-\delta$ (idler) frequencies above and below $\omega_c/2$, where ω_c is the pump frequency.

(a) For an interaction Hamiltonian of the form,

$$\hat{H}_{\mathrm{int}}(\delta) = i\hbar\chi\beta\left[\hat{a}^{\dagger}_{\omega_c/2+\delta}\hat{a}^{\dagger}_{\omega_c/2-\delta} - \hat{a}_{\omega_c/2-\delta}\hat{a}_{\omega_c/2+\delta}\right], \quad (1)$$

derive the relations for the output signal and idler field operators in terms of the corresponding input (vacuum) field operators. Here β is the (real) pump field amplitude and χ is the nonlinear coefficient associated with the nonlinear crystal used. Assume some interaction time t, and write your answers in terms of an overall squeezing parameter $k = \chi\beta t$. Note that there is an interaction term for every frequency offset δ from $\omega_c/2$.

(b) Assuming that the *total* input field at the input signal port to the BHD is given by the Heisenberg operator, $\hat{a}(t) = \int \mathrm{d}\omega_k \hat{a}_{\omega_k}(0)e^{-i\omega_k t}$, derive the Fourier operator of the BHD photocurrent $\hat{I}_-(\omega)$ and show that it depends linearly on Schrödinger continuum operators $\hat{a}_{\omega_{\mathrm{LO}}+\omega}(0)$ and $\hat{a}^{\dagger}_{\omega_{\mathrm{LO}}-\omega}(0)$. Here ω_{LO} is the LO frequency which we will choose to be equal to $\omega_c/2$, ω_k ranges over positive frequencies around $\omega_c/2$, ω is an electronic frequency within the bandwidth of the BHD receivers, and $\hat{a}_{\omega_k}(0)$ is a Schrödinger operator in the continuum with the usual continuum commutation relations.

(c) Derive the single-sided power spectral density (PSD), $S_{I_-}(\omega \geq 0) \equiv S_{I_-I_-}(\omega) + S_{I_-I_-}(-\omega)$, in terms of the two-point correlators of the output field operators of the PDC nonlinear crystal in the Fourier domain.

(d) Using the relations between the vacuum input and the squeezed output field operators of the PDC nonlinear crystal found in part (a), rewrite the single-sided power spectral density of part (c) in terms of vacuum two-point correlators in the Fourier domain.

(e) Evaluating the vacuum two-point correlators, show that the single-sided power spectral density of the (ideal) BHD detector for such a squeezed input is given by,

$$S_{I_-}(\omega \geq 0) = 4\pi\bar{\dot{N}}\left[\cosh(2k) - \cos(2\phi_{\mathrm{LO}})\sinh(2k)\right], \tag{2}$$

where $\bar{\dot{N}}$ is the average photon flux in the LO and ϕ_{LO} is the LO phase. Notice that the broadband squeezing found in the BHD photocurrent (or anti-squeezing) is due to the correlations found in the continuum of signal and idler modes.

(f) Sketch $S_{I_-}(\omega \geq 0)$ for $k = 0$ (no squeezing, corresponding to the vacuum noise level or the standard quantum limit (SQL)), and for $k \neq 0$ with LO phases $\phi_{\mathrm{LO}} = 0$ and $\phi_{\mathrm{LO}} = \pi/2$.

(g) If the detectors used in the BHD receiver do not have unit efficiency (i.e., $\eta_{\mathrm{QE}} < 1$), the amount of measured squeezing in the BHD photocurrent will be reduced below that of the actual squeezing in the optical field. One can think of this as arising due to vacuum noise leaking into the detector input and diluting the squeezed fields. Rederive $S_{I_-}(\omega)$ in terms of $\eta_{\mathrm{QE}} < 1$, and find the minimum noise level of the BHD (for any ϕ_{LO}) below the SQL in terms of k and η_{QE}.