Applied Physics 190c: Homework #6

(Dated: May 18th, 2017)

Due: Friday, May 26th

1. **Reading:** Section V of Clerk2010 [1], Paper I [2], Paper II [3], and Paper III [4].

2. **(10 points) Getting going with the Quantum Optics Toolbox**

   (a) Install on your computer the Quantum Optics Toolbox, a set of matlab functions written by Sze M. Tan at the University of Auckland for the simulation of quantum systems. Alternatively, install the more modern version of this toolbox, Qu-Tip, which is based upon Python.

   (b) Read the accompanying documentation (qousersguide.pdf) entitled, "A Quantum Optics Toolbox for Matlab 5."

   (c) Implement a script that reproduces Figure 1 from the users guide.

3. **(60 points) Quantum measurement in practice**

   We have learned a lot of different quantum optics techniques, and now it is time to see if we can apply them to cutting-edge research problems. Below you will find descriptions of three different papers that I have chosen from the recent literature. Pick just one of them, and rederive the important results from the paper.

   Paper I: "Back-action evasion and squeezing of a mechanical resonator using a cavity detector" [2]. This paper is by Aash Clerk, Florian Marquardt, and Kurt Jacobs. This paper deals with a cavity-optomechanical system in which the mechanical motion of a resonator is dispersively coupled (what we called parametric coupling) to the optical cavity frequency. We saw in class that using a single laser frequency on resonance with the optical cavity to probe the mechanical motion resulted in a quantum limit to the imprecision-back-action-force product, $\sqrt{S^I_{xx} S_{FF}} \geq \hbar/2$. The net result of this is that there is an optimal power (called the SQL power) at which point one reaches a minimum in the added measurement noise. This is called the standard quantum limit (SQL) and it equals exactly the zero-point fluctuation noise of the mechanical resonator in the ideal-detector limit. For optical powers below the SQL power the imprecision due to shot noise on the light dominates, whereas for optical powers above the SQL power the Stochastic back-action force due to optical shot noise dominates and drives the mechanics.

In the Clerk, et al., paper [2], they describe a method utilizing two lasers of different frequencies to simultaneously probe the mechanical motion. If the lasers have a frequency difference that is twice the mechanical frequency then the claim is that one can avoid the effects of shot-noise back-action in the read-out of the mechanical motion. Go through the paper, and derive for yourself the important relations that lead to this conclusion. Pay special attention to any approximations and required post-processing of the measurement signals. Provide a physically intuitive explanation for this result.

Paper II: "Enhanced Quantum Nonlinearities in Two-Mode Optomechanical System" [3]. This paper is by Max Ludwig, Amir H. Safavi-Naeini, Oskar Painter, and Florian Marquardt. This paper describes methods to realize continuous quantum non-demolition (QND) measurements of the energy or phonon number in a mechanical resonator. As mentioned in class, under certain conditions this two-mode set-up can be used to perform QND measurements of the photon number in one of the cavities as well.

Derive the main results of this paper. Comment on the applicability of this method for QND measurements, and under what conditions the QND-nature of the measurement breaks down. Utilizing the Quantum Optics Toolbox attempt to calculate the quantum trajectories shown in Figure 3(b) and (d) from a numerical simulation of the quantum master equation. You can also use other numerical software such as Qu-Tip which runs on Python. You will have to do some reading about Quantum Monte Carlo methods (a useful description is given in the quantum optics toolbox user's guide, but you may want to look up more pedagogical sources such as Ref. [5]). You may also want to consult Max Ludwig's thesis and our recent arXiv paper on realizing such a two-moded system in photonic crystals. Even if you don't get all the way there, you can show your work in the form of what you learned and whatever simple examples you were able to realize.

Paper III: "Observing single quantum trajectories of a superconducting quantum bit" [4]. This paper is by K. W. Murch, S. J. Weber, C. Macklin, and I. Siddiqi, and it explores the measurement of the quantum trajectory of a superconducting qubit. This is made possible by the fact that the qubit dominantly couples to a single mode of a superconducting electromagnetic cavity (the “environment”), and that one can measure with quantum-limited precision the state of this electromagnetic cavity using a microwave parametric amplifier.

First, describe the details of the measurement scheme employed. For the second part you have two choices: either (i) dig further into the technology behind the microwave parametric amplifiers used in this work, providing a detailed analysis of the state-of-the-art of this technology, or (ii) try and numerically model the measurements presented in Figure 3 of Ref. [4]. Again in this case you will need to use software such as the Quantum Optics Toolbox or Qu-Tip, and you will likely want to read up on Quantum Monte Carlo methods [5].

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[1] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, arXiv:0810.4729 (2010).

[2] A. A. Clerk, F. Marquardt, and K. Jacobs, New J. Phys. **10**, 095010 (2008).

[3] M. Ludwig, A. H. Safavi-Naeini, O. Painter, and F. Marquardt, Phys. Rev. Lett. **109**, 063601 (2012).

[4] K. W. Murch, S. J. Weber, C. Macklin, and I. Siddiqi, Nature **502**, 211 (2013).

[5] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998).